Using the Dirichlet distribution
to specify a random effect on a transition probability matrix

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Abstract: Inter-state transition probability matrices are a common feature of state-space and Markov chain models, which are commonly applied to model animal movements or animal behaviour. In many cases, it may be desirable to include a random effect (of, for example, individual or species) on the transition matrix entries. We propose a simple method for specification of such random effects using the Dirichlet distribution. We detail model specification and maximum likelihood estimation via a case study, in which a Dirichlet random effect accounts for inter-whale variability in a Markov chain model for sperm whale dive-type transitions. The data are time series data on the dive behaviour of 12 sperm whales, where dives have been pre-classified into 11 types by experts. We specify a random effect of individual w on the K-dimensional transition matrix P (for the case study, $K = 11$ and w ranges from 1-12). To do so, we let row i of the individual-specific transition matrix P_w be a draw from a Dirichlet distribution with parameters $\vec{\alpha_i} = (\alpha_{i1}, \alpha_{i2}, ...\alpha_{iK})$. Then the inter-individual variability is measured by $\alpha_{i0} = \sum_{k=1}^{K} \alpha_{ik}$ (with larger α_{i0} indicating less variability), and the population-average transition probability $P(i, j)$ is given by $\frac{\alpha_{ij}}{\alpha_{i0}}$. This formulation results in an analytically tractable likelihood expression, subject to quick, efficient maximisation. The ease of maximum likelihood estimation also facilitates model selection (using, for example, AIC). Application of the method to the case study data provides new biological insight, firstly by quantifying inter-individual variation in sperm whale dive behaviour. Moreover, the dataset includes behaviour data with and without acoustic disturbance, and a random-effect model facilitates more robust measurement of whales' behavioural response to the disturbance. Analogous methods could be used for easy specification of a random effect on any probability mass function.